# REPORT No. 212

## STABILITY EQUATIONS FOR AIRSHIP HULLS

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### SUMMARY

In the text are derived simple formulæ (9), (13), for determining, directly from the data of wind tunnel tests of a model of an airship hull, what shall be the approximate character of oscillation, in pitch or yaw, of the full-scale ship when slightly disturbed from steady forward motion.

### OBJECT

It is desired to write the equations of motion for a finned but carless airship, slightly disturbed from swift head-on steady translation, and thence to deduce criteria for stability in pitch and yaw. These criteria may be expressed first in terms of full-scale coefficients; then in terms of model coefficients applied to full-scale craft. Incidentally some other features of the motion will be noticed.

The present treatment, slightly modified, was prepared for the Bureau of Aeronautics, June 30, 1922, and by it was submitted for publication to the National Advisory Committee for Aeronautics.

### INITIAL CONDITIONS

To simplify the treatment, assume the hull symmetrical about its long axis; the centers of mass and volume practically coincident; the weight annulled by the buoyancy; the controls neutral; the motion head-on through still air. The second assumption evades the gravity moment, which for a hull at high speed usually is small compared with the tail moment, and is balanced with the stabilizer.

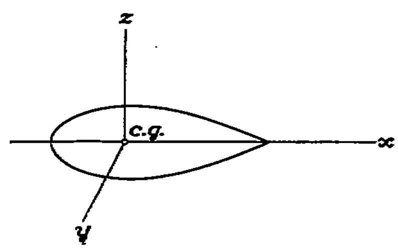


FIG. 1.—Reference axes for airship hull

Starting from the centroid, Figure 1, the axes are $x$ running aft, $y$ aport, $z$ upward. In general, the increments of velocity along and about these axes are, respectively, $u, v, w, p, q, r$; the increments of the air force and moment are $X, Y, Z, L, M, N$.[1] Initially all the components of velocity and wrench are zero, except the forward speed $U$ and the resistance $X_0$, which latter has a negligible, if any, effect on the movement other than straightforward.

### MOTION IN SIMPLE PITCH

If now the ship with steady speed $U$ is given a slight pitch $\theta$, the entire system of external forces other than $X_0$ is equivalent to the normal air force $Z$ at the centroid, and the pitching moment $M$ about it. Hence, by d'Alembert's principle, the conditions for kinetic equilibrium are

$$Z=m\dot{w}_1=m(\dot{w}-U^{\text{[2]}}q) \qquad (1)$$

$$M=B\dot{q} \qquad (2)$$

[1] Unless otherwise stated, all components are referred to $x, y, z$, fixed in the hull and moving with it.

[2] If $\theta$ is not small, $U$ in these equations should be replaced by $u_0=U\cos\theta$. The difference is immaterial for $\theta<5°$.

where $m$ is the effective mass of the ship; $B$ its moment of inertia about $y$; while the space acceleration of the centroid is $\dot{w}_1$ if referred to the $z$ axis as instantaneously fixed in space direction, but $\dot{w}-Uq$ if referred to $z$ moving with the ship.[3] If $m'$ is the actual mass of the ship, the effective mass $m$, that is, the ship's actual mass plus its virtual mass due to accelerating the fluid, can be taken as about 1.5 $m'$ for transverse accelerated motion whether normal or lateral.

Since both static and damping forces, due to $\theta$ and $q$, are present, it is advantageous to place in evidence the components of $Z$ and $M$. The part of $Z$ due to $\theta$ is $\theta Z_\theta$, where $Z_\theta$ denotes $\partial Z/\partial\theta$; the part due to $q$ is $qZ_q$. They are the usual lift and the damping force; and compose practically the whole of $Z$. Similarly the static and damping parts of $M$ are $\theta M_\theta$, $qM_q$. Substituting these four components in (1), (2), gives

$$\theta Z_\theta+qZ_q=m(\dot{w}-qU) \qquad (3)$$
$$\theta M_\theta+qM_q=B\dot{q} \qquad (4)$$

## CONDITIONS FOR STABILITY IN PITCH

The angular velocity is constant for $\dot{q}=O$, and declines on addition of some restoring moment, due, for example, to the lowering of the centroid. Making $\dot{q}=o$ reduces (4) to

$$\theta M_\theta+qM_q=O \qquad (5)$$

Also for such speeds that $\dot{w}$ is small compared with $qU$, one can write (3) more simply

$$\theta Z_\theta+q(Z_q+mU)=O \qquad (6)$$

These two equations are simultaneous, and taken together represent approximately the conditions essential for dynamically stable motion, i. e., for unamplifying pitch, expressed in familiar aerodynamic quantities.

By (5) the damping moment $qM_q$ rotationally balances the disturbing moment $\theta M_\theta$; by (6) the disturbing force $\theta Z_\theta$, the damping force $qZ_q$, and the inertia force $qmU$, are in translational balance.

## CRITERION OF PITCH STABILITY

In (5) and (6) the variables $\theta$ and $q$ are the angular displacement and velocity of the small stable oscillation; the other six quantities are specified constants. Eliminating $\theta$, $q$, gives the stability condition, viz, the relation between these constants which is necessary to satisfy (5), (6), for all small values of $\theta$ and $q$

$$\frac{Z_\theta\ M_q}{M_\theta(Z_q+mU)}=1 \qquad (7)$$

This criterion can also be written in terms of model data. If $s$ is the scale ratio, $u$ the model speed, and if primes mark the model symbols, the values in (7) are related to the model values thus:

$$\left.\begin{aligned} Z_\theta/M_\theta&=\frac{1}{s}Z'_\theta/M'_\theta \\ M_q/U&=s^4\mu/u \end{aligned}\right\} \qquad (8)$$

where $\mu$ is the coefficient of damping moment in pitch for the model with head-on speed $u$. One may write $Z_q+mU=nmU$, where $n$ is to be found experimentally, since $Z_q\propto U$. Putting these new values in (7), the working criterion for pitch becomes

$$a\frac{\mu}{u}\frac{Z'_\theta}{M'_\theta}=1 \qquad (9)$$

where $a=\frac{s^3}{mn}$. The stability criterion (9) is now in a form convenient to use with familiar wind tunnel data.

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[3] Books on mechanics prove $\dot{w}_1=\dot{w}-Uq$. See Wilson's Aeronautics §48.

### CRITERION OF YAW STABILITY

By a very similar process the yaw velocity proves to be constant when

$$\psi N_\psi + r N_r = 0 \qquad (10)$$

$$\psi Y_\psi + r(Y_r - m U) = 0 \qquad (11)$$

and decreases when the damping exceeds the disturbing moment; that is when

$$\frac{Y_\psi \; N_r}{N_\psi \; (Y_r - m U)} > 1 \qquad (12)$$

Hence

$$a \frac{\mu}{u} \frac{Y'_\psi}{N'_\psi} > 1 \qquad (13)$$

is the working criterion for yaw stability, and is the analogue of (9). Here $a = \frac{s^3}{mn'}$, where $n' = (Y_r - m U)/m U$, as previously explained for pitch; and the primes indicate model symbols.

By (10) the damping moment $rN_r$ rotationally balances the disturbing moment $\psi N_\psi$; by (11) the disturbing force $\psi Y_\psi$, the damping force $rY_r$, and the inertia force $-rm U$, are in translational balance.

If the yawing velocity is accelerated, and $C_r$ is the opposing moment due to angular inertia, (10) becomes

$$\psi N_\psi + {}_r N_r = C_r \qquad (14)$$

which is the analogue of (4), and is simultaneous with (11).

### ALTERNATIVE DERIVATIONS

The foregoing treatment fixes the attention upon the chief forces and moments governing the ship's motion under the simple conditions therein assumed. Crocco in 1907, and various British writers some years later, investigated the motion of full-rigged airships with their centroids sufficiently below their buoyancy centers, and obtained approximate criteria which are reducible to forms (9), (13). It is therefore unnecessary to discuss here the practical applicability of the criteria, since that has been done in the writings just cited. Reference may be made to R. & M. Nos. 257 and 361 of the National Physical Laboratory of England.

In passing one may observe that British writers give as stability criteria in pitch and yaw

$$M_q Z_w / M_w (Z_q + U) = 1 \qquad (7_1)$$

$$Y_\psi N_r / N_\psi (Y_r - U) > 1 \qquad (12_1)$$

in which the symbols are for the ship. On substituting for these symbols their values (in the forms given by those writers) in terms of the model symbols, one readily obtains (9) and (13).

### MOMENT ARMS IN YAW

In (12) one may write $N_\psi / Y_\psi = x_1$, which is the arm of the disturbing force, or its distance from the centroid. Hence the other factor must be a length, say $N_r/(Y_r - m U) = x_2$, where $x_2$ is the distance from the centroid to the resultant of $rY_r$ and $-mr U$. By (12) therefore

$$x_2 / x_1 = 1 \qquad (15)$$

for steady yawing motion. This relation is obvious without the aid of algebra; for (15) merely implies that the lateral disturbing force $\psi Y_\psi$ is equal and opposite to the resultant $r\,(Y_r - m U)$ of the reacting forces which kinetically balance it.

In like manner $N'_\psi / Y'_\psi$ is the arm of the disturbing force on the model; hence the other factor $a\mu/u$ in (13) must be the arm of the reacting forces. By dynamical similarity the arms of the forces on the ship are $s$ times as long as those of the model; but equal them when all are expressed as percentages of the length of the major axis of the respective hulls. The stability criterion in either case is the ratio of the reacting to the disturbing arm.